Allocating Network Capacity

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Abstract

The paper examines efficient mechanisms to allocate scarce transmission capacity when the upstream generation market is not competitive. I analyze an environment in which the regulator owns the transmission infrastructure. The regulator cannot directly intervene in the generation market. However, through the allocation mechanism, it can affect incentives of the upstream generators, thereby modisying the outcome of the generation market.

I find that, when the allocation is related to the generated output, the mechanism can partly correct the standard production externality intrinsic in the transmission market (determined by the fact that the constraint on the transmission capacity is on the net, rather than on the gross, flow of energy).

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1 Introduction

This paper is part of a project that analyzes a market characterized by two nodes. In each of these nodes there is a given number of consumers and a certain number of upstream firms, all producing an homogenous commodity. In order for producers located in one node to reach consumers located in the other node, a downstream interconnection infrastructure - be it a transmission infrastructure in the electricity case, a pipeline, a bridge, a railroad - is essential. The terms upstream and downstream refer to the physical sequence of production, in which transportation follows production. However, from a purely economic perspective, interconnection capacity is an input for the upstream firms.

In the present environment, a regulator owns the interconnection infrastructure, and has to allocate it among the various upstream producers. The regulator's objective is not only to implement an efficient allocation mechanism (which maximizes the rent for the interconnection system's owner) but also, to implement the mechanism that maximizes welfare in the commodity market. The first best would be achieved when *both* efficiency in the upstream market (achieved under marginal cost pricing) and efficiency in the allocation mechanism are achieved. Efficiency in the allocation of interconnection prescribes that a firm charging a lower price (which, under perfect competition is equivalent to a more efficient firm) has priority in the transit over a firm charging a higher price (less efficient under perfect competition).

When attaining the first best is infeasible, the regulator has to sacrifice efficiency in one of the two markets (upstream and interconnection) in favor of the other.

Given the allocation system announced by the regulator, producers compete in quantities, without being affected by any additional regulatory constraints. The regulator's task consists of modifying the incentive schemes of the producers through the interconnection capacity allocation, thereby enhancing welfare. That is, the incentive scheme induced by the interconnection allocation mechanism should aim at increasing the market aggregate output above the Cournot level - which would prevail in the absence of any regulatory interventions - bringing it closer to the competitive quantity.

Regulation of the essential facility alone, coupled with competition in the upstream sector, is common in many network industries. Sometimes, it is the regulator (or the State) that directly owns the essential facility in some industries, including electricity, gas, railways, cable-TV, water etc. Often, in these industries, the upstream sector is characterized by oligopolistic competition.

The literature on interconnection capacity is mostly applied to electricity markets. Their scope differs from that of the current paper, in that, most of them analyze rent-maximizing allocation mechanisms. Joskow and Tirole (2000), for example, analyze how two rent-maximizing (in a perfectly competitive environment) allocation schemes, such as financial and physical transmission rights, perform in an imperfectly competitive environment. Borenstein, Bushnell, and Stoft (2000) analyze the competitive impact of additional transmission infrastructure. Their findings are the following. First, there may be no relationship between the effect of a transmission line in spurring competition, and the actual electricity that flows on the line in equilibrium. Second, limited transmission capacity can give a firm the incentive to restrict its output in order to congest transmission into its area of dominance. Third, relatively small investments in transmission may yield surprisingly large payoffs in terms of increased competition. Along this line, Stoft (1999) analyzes financial transmission rights applied in a generation market characterized by Cournot competition.

The current paper is similar to the above mentioned papers, in the sense that the analyzed environment reproduces institutional details that are typical of the electricity sector. In particular, I assume that, due to physical reasons, the interconnection is characterized by an upper bound on the *net* flow. That is, the flows occuring in opposite directions must not differ by more than the constraint. However, the present work differs from the previous literature, since it assumes that interconnection capacity is not scarce. The two relevant nodes are always fully interconnected, and the cost of interconnection varies. Full interconnection allows me to consider an individual market, thereby greatly simplifying the model. In this context, I characterize an allocation scheme maximizing welfare in the overall commodity (electricity) market. At the same time, this work is similar to the above mentioned literature in the sense that, it refers to a situation in which the interconnection is characterized by an upper bound on the *net* flow.

The structure of the paper is as follows. Section 2 introduces the model, section 3 provides the results, while section 4 concludes.

2 The model with one firm per node

This model stylizes the properties of the interconnection capacity when the constraint on the flow is net, the case in electricity transmission lines. In this case, the transfer cost is paid by firms in the node whose aggregate export exceeds aggregate import.

Suppose there are two nodes. The aggregate demand in the two nodes is of the form $P(Q) = A - bq$. Each firm produces both for its domestic market, and for the other node (we will call it "export"). If import equals export in each of the nodes, firms do not need transmission capacity, as long as there exists an infinitesimal amount of available transmission capacity. If, on the other hand, in any node, import and export differ, then the firm in the node whose export exceeds import needs an amount of transmission capacity exactly equal to the difference between the exported and the imported quantities.

Firms face an unlimited amount of transmission capacity, available through an *indirect* line that takes a long loop before actually connecting the two nodes. Through this process, part of the production is lost. This raises the cost for the producers by a large amount. This assumption rules out the possibility that the two markets are separated: whatever the output produced by each of the firms, there is a single market aggregating the demands and the productions in the two nodes, with a single price. Assume for the sake of simplicity that this single price results from an institutional rule in place in the market under consideration. This rule specifies that, in the absence of active transmission constraints that limit the flow of electricity between the two markets, prices in the them have to be the same. This institutional rule ignores the costs associated to losses in the trasnmission line; in other words, it ignores the transportation costs.

Under a broad range of circumstances, interconnection has a competitive effect; hence, firms with market power do not find it profitable to build it. In this case, however, since the market is fully connected, regardless of the capacity of the *direct* transmission line, firms may find it convenient to actually build the direct transmission line. The direct transmission line entails a building cost; however, it then reduces the losses (hence, it reduces the cost). I assume that firms always find it more convenient to build a direct transmission line, rather than exploiting the available indirect line. The properly discounted cost of building the line is always less than the extra costs the firm incurs when it exploits the indirect transmission capacity.

Thus, I have established that, when firms located in one node find it optimal to export more than they import, they optimally build the necessary capacity to accomodate their decisions.

I now introduce a second available direct line, owned by the regulator, and managed according to welfare-maximizing criteria. This line directly links the two nodes. Hence, there is no substantial transmission cost associated to the usage of this line. The regulator allocates the capacity of the transmission line it manages among the various generators.

Hence, in order to obtain a given amount of transmission capacity, a firm has two alternatives. Either it builds its own transmission line, and uses it, or it uses the transmission line owned by the regulator, if the conditions established by the regulator yield a higher profit than the individual investment.

In such a setting, a particular strategic environment is generated, in which, when import and export differ in each node, a marginal increase in export by the low-export firm has two effects on the high-export firm.

First, it decreases the rival's marginal revenue (the standard Cournot effect). Second, it decreases its transportation cost. In other words, in spite of the two products being perfect substitutes, a marginal increase of the output of the firm characterized by the lowest export generates a *positive externality on the rival's cost.* If the positive externality on the rival's cost exceeds the negative externality on the rival's revenue, then the marginal increase of export by the low-export firm may induce an overall positive externality on the high-export rival.

The peculiarity of the strategic environment may be summarized as follows: as long as a marginal increase in output by the low-export firm generates a positive externality on the high-export rival, the high-export firm regards the two products as strategic complements while the low-import firm regards them as strategic substitutes.

Given that it is assumed that saturation of the transmission line never occurs, the environment I have in mind is captured by the following model.

There are two firms, i and j, producing a homogenous product, and selling it into a single market. The firms' cost function is made up of two components: a) an *individual* component that depends on the output produced by each firm, equal to c_i and c_j per unit produced respectively, b) a cost incurred by the firm only if it produces more than its rival. This second component, assumed to be same for both the firms, is denoted by the letter c. c is the cost of building the additional transmission line, per unit of capacity (so that the total cost of building k units of capacity is ck)[1]. Assume, without loss of generality, that i is weakly more efficient than j, ($c_i \leq c_j$). The individual component of the cost fucntion is the per-unit generation cost, while the common component represents the cost of building the necessary transmission infrastructure for the firm. Notice that, in this model, the allocation of the regulator's transmission line is equivalent, from the firm's viewpoint, to a reimbursement of part of its costs. When firm i receives no reimbursement from the regulator (i.e., it is not allocated the regulator's transmission capacity), the cost function for firm i takes the following form:

$$C_i(q_i) = \begin{cases} c_i q_i + c(q_i - q_j) \ \ if \ q_i > q_j \\ c_i q_i \ otherwise \end{cases}$$

Each firm perfectly observes the rival's cost, and the two contestants play a game of perfect information, with the following timing:

First, the regulator announces the allocation mechanism. Second, producers compete à la Cournot (knowing that reimbursement is the only possible form of intervention by the regulator in the market). Third, reimbursement is awarded.

Market demand is given by $p(Q) = A - bQ$. The regulator can reimburse the common component of the cost c up to a maximal aggregate amount of t. t has to be allocated between i and j, and the regulator has to decide the reimbursement criterion in order for social welfare to be maximized. The regulator does not receive money in exchange for the allocation. The (standard) reason for this is that inducing an increase in equilibrium aggregate output yields a higher social welfare than simply appropriating the rent of the firm. The awarded reimbursement, denoted $t_i(q_i, q_j)$ and $t_j(q_i, q_j)$ respectively for firm i and for firm j, becomes then solely a function of firms' production. The regulator faces the following constraints, in its allocation criterion:

1) $t_i + t_j \leq t$;
2) $0 \leq t_i \leq \max(0, c(q_i - q_j))$;
3) $0 \leq t_j \leq \max(0, c(q_j - q_i))$

The simplified version of the model allows to consider a single decision variable for each firm, instead of two (output in the two different markets), while it preserves the two relevant features of the original scenario:

[1] Here, a dynamic environment is being approximated by a static model. The investment in the interconnection line evidently brings about benefits in more than one period.

First, the fact the a portion of the cost is paid only by the high-output firm, and only on the difference between the two firms' output. Second, that the high-production firm regards its own product and the rival's as strategic complements, while the low-production firm regards its own product and the rival's as strategic substitutes.

2.1 The efficiency benchmark

Social welfare is determined as:

$$\max_{q_i,q_j} \frac{Q\left(A-\left(A-bQ\right)\right)}{2}+Q\left(A-bQ\right)-C\left(Q\right)$$

where Q is the aggregate output produced in the two nodes.

In order to obtain the characterization of the social efficiency benchmark, we proceed through the characterization of the overall market cost function $C\left(Q\right)$. In general, the cost function results from the aggregation of the cost functions of the individual firms operating on the market. However, in this specific case, each firm's cost is a function of both firms' output. Hence $C\left(Q\right)=C\left(q_i,q_j\right)$. In order to properly characterize the market's cost function, I claim that, if $q_i=q_j$, then the market's cost function is given by $C\left(Q\right)=\left(\frac{c_i+c_j}{2}\right)Q$. If, on the other hand, $Q=q_i$, then the cost function is $C\left(Q\right)=\left(c_i+c\right)Q$. We can then characterize the following:

Lemma 1 *The market aggregate cost function is given by* $Q\left(\min\left(\frac{c_i+c_j}{2},c_i+c\right)\right)$.

Proof. First, notice that an implication of the lemma is that the cost-minizing supply schedule prescribes either $q_i^*=q_j^*$, or $q_j^*=0$. Suppose instead, that at a generic Q', $q_i'>q_j'>0$, $q_i'+q_j'=Q'$.Then

$$C\left(q'\right)=\left(c_i+c_j\right)q_j+\left(c_i+c\right)\left(q_i-q_j\right)$$

At an alternative schedule prescribes $q_i''=q_i'-\epsilon$, $q_j''=q_j'+\epsilon$, $q_i''+q_j''=Q'$, ϵ units cost $\frac{c_i+c_j}{2}$ instead of c_i+c. If $\frac{c_i+c_j}{2}<c_i+c$, then the alternative supply schedule yields a lower cost.

Alternatively, assume $q_i''=q_i'+\epsilon$, $q_j''=q_j'-\epsilon$, $q_i''+q_j''=Q'$. Then, ϵ units cost c_i+c instead of $\frac{c_i+c_j}{2}$. If $c_i+c<\frac{c_i+c_j}{2}$, then this supply schedule yields a lower cost than the original $\left(q_i',q_j'\right)$. Hence, there are no cases in which $q_i'>q_j'>0$ can be the cost-minimizing supply schedule.

If $\frac{c_i+c_j}{2} < c_i + c$, then $C(Q) = \left(\frac{c_i+c_j}{2}\right) Q$, and the cost-minimizing supply schedule prescribes $q_i = q_j$; if $\frac{c_i+c_j}{2} > c_i + c$, then $C(Q) = (c_i + c) Q$. Following the lines of the first part of the proof, it is easy to show that, for any given Q, reshuffling the production among the two firms increases the cost with respect to the cost-minimizing schedule (which depends, as just shown, on the cost parameters). ■

It is now immediate to characterize the social optimum as the point in which demand equals marginal cost.

Lemma 2 *First-best social efficiency prescribes* $p = \min\left(\frac{c_i+c_j}{2}, c_i + c\right)$, *and* $Q = \frac{A-\min\left(\frac{c_i+c_j}{2}, c_i+c\right)}{b}$

2.2 The duopoly outcome: unregulated

In this strategic setting, while the firm with the lowest production is affected by the marginal increase of quantity of its rival (the standard negative externality), the firm with the highest production is positively affected (in its cost structure) by an increase in the rival's production. In order to ensure that each firm produces a strictly positive output, I assume that $c_j < A + c$.

Firm i's optimization problem is the following:

$$\max_{q_i} (A - bq_i - bq_j) q_i - \begin{cases} c_i q_i - c(q_i - q_j) \ if \ q_i > q_j \\ c_i q_i \ if \ q_i \leq q_j \end{cases}$$

The reaction functions take the following form:

$$R_i(q_j) = \begin{cases} \min\left(\frac{(A-c_i-bq_j)}{2b}, q_j\right) \ if \ \pi_{i,q_i^* \leq q_j^*} > \pi_{i,q_i^*>q_j^*} \\ \max\left(\frac{(A-c_i-c-bq_i)}{2b}, q_j\right) \ if \ \pi_{i,q_i^* \leq q_j^*} > \pi_{i,q_i^*>q_j^*} \end{cases}$$

I restrict myself to the case in which $q_i^{*UC} > q_j^*$, where q_i^{*UC} denotes the optimal outcome in the currently analyzed unregulated Cournot situation, which turns out to be an equilibrium condition. Hence, i's reaction function, for $q_i \geq \frac{(A-c_j)}{3b}$, is the following:

$$R_i(q_j) = \begin{cases} q_j \ if \ \pi_{i,q_i^*=q_j^*} > \pi_{i,q_i^*>q_j^*} \\ \max\left(\frac{(A-c_i-c-bq_j)}{2b}, q_i\right) \ if \ \pi_{i,q_i^* \leq q_j^*} > \pi_{i,q_i^*>q_j^*} \end{cases}$$

$$q_j = \left(\frac{(A - c_j - c - bq_i)}{2b} \right)$$
$$q_i = \left(\frac{(2A - 2c_j - 2c - 4bq_j)}{2b} \right)$$

In principle, there are three possible outcomes:
i) $q_i^* > q_j^*$;
ii) $q_i^* < q_j^*$;
iii) $q_i^* = q_j^*$.

Lemma 3 *In equilibrium, it cannot be that* $q_i^{*UC} < q_j^{*UC}$.

Proof. Assume $q_i^{*UC} < q_j^{*UC}$. Then, the reaction functions are:

$$R_i(q_j) = \frac{(A - c_i - bq_j)}{2b}$$

$$R_j(q_i) = \frac{(A - c_j - c - bq_i)}{2b}$$

This entails equilibrium values given by:

$$q_i^* = \frac{A - 2c_i + c_j + c}{3b}$$
$$q_j^* = \frac{A - 2c_j + c_i - 2c}{3b}$$
$$p = \frac{A + c_j + c_i + c}{3}$$

$$q_j^{*UC} > q_i^{*UC} \Leftrightarrow c_j + c < c_i$$

However, this is a contradiction of i being more efficient than j. ■

Since $C_i(q) \leq C_j(q)$ for $q_i \leq q_j$, there does not exist an equilibrium in which i produces less than j. We are then left with case i), and iii).

Lemma 4 *If* $c_i < c_j - c$, *then in equilibrium* $q_i^{*UC} > q_j^{*UC}$

Proof. Assume $q_i^* > q_j^*$. Then, the reaction functions are:

$$R_i(q_j) = \frac{(A - c_i - bq_j - c)}{2b}$$

$$R_j\left(q_i\right)=\frac{\left(A-c_j-bq_i\right)}{2b}$$

It follows that in equilibrium:

$$q_i^*|q_i^*>q_j^*=\frac{A-2c_i+c_j-2c}{3b}$$

$$q_j^*|q_i^*>q_j^*=\frac{A+c_i-2c_j+c}{3b}$$

The set of values for which the previous equilibrium is feasible (i.e, it is internally coherent) is given by q_i^* given that $q_i^*>q_j^*$ and q_j^* given that $q_i^*>q_j^*$ such that q_i^* given that $q_i^*>q_j^*$ exceeds q_j^* given that $q_i^*>q_j^*$, and namely by $c_j>c_i+c$

$$q_i^{*UC}=\frac{A-2c_i+c_j-2c}{3b}$$

$$q_j^{*UC}=\frac{A+c_i-2c_j+c}{3b}$$

For the values at which the equilibrium is feasible, we need to check there does not exist any profitable deviations.

I first rule out deviations to $q_j^*<q_i'<q_i^*$. Indeed, we know $q_i'\notin\arg\max\pi_i|q_i^*>q_j^*,q_j^*=R_j\left(q_i^*\right)$. Hence, deviating to $q_j^*<q_i'<q_i^*$ cannot be profitable for firm i.

I now rule out deviations to $q_i''=q_j^*$. If $q_i''=q_j^*=\frac{A+c_i-2c_j+c}{3b}$, $p=\frac{A-2c_i+4c_j-2c}{3}$, and

$$\pi_{i,q_i^*=q_j^*}=\left(\frac{A+c_i-2c_j+c}{3b}\right)\left(\frac{A-5c_i+4c_j-2c}{3}\right)=$$

$$=\frac{-4Ac_i+2Ac_j-7cc_i+8cc_j-5c_i^2-8c_j^2+14c_ic_j-Ac+A^2-2c^2}{9b}$$

The deviation profit has to be compared with the equilibrium profit. When $q_i^{*UC}>q_j^{*UC}$, $q_i^{*UC}=\frac{A-2c_i+c_j-2c}{3b}$, $q_j^{*UC}=\frac{A+c_i-2c_j+c}{3b}$, $p=\frac{A+c_i+c_j+c}{3}$, the profit accruing to firm i is given by:

$$\pi_{i,q_i^*>q_j^*} = \left(\frac{A - 2c_i + c_j - 2c}{3b}\right)\left(\frac{A - 2c_i + c_j + c}{3}\right) - c\left(\frac{c_j - c_i - c}{b}\right) =$$

$$= \frac{-4Ac_i + 2Ac_j + 11cc_i - 10cc_j + 4c_i^2 + c_j^2 - 4c_ic_j - Ac + A^2 + 7c^2}{9b}$$

As long as $c_j > c_i + c$, the deviation is never profitable. Indeed, $\pi_{i,q_i^*=q_j^*} < \pi_{i,q_i^*>q_j^*}$, implies:

$$18c\left(c_j - c_i\right) - 9c_i^2 - 9c_j^2 + 18c_ic_j - 9c^2 < 0$$

which never holds. ■

The following is true

Lemma 5 *If $c_j < c_i + c$, then in equilibrium $q_i^{*UC} = q_j^{*UC}$*

Proof. The equilibrium output is given by:

$$q_i^{*UC} = q_j^{*UC} = \frac{(A - c_j)}{3b}$$

It follows that $p = \frac{A+2c_j}{3}$, and

$$\pi_{i,q_i=q_j} = \left(\frac{A - c_j}{3b}\right)\left(\frac{A + 2c_j - 3c_i}{3}\right) =$$

$$= \frac{-3Ac_i + Ac_j - 2c_j^2 + 3c_ic_j + A^2}{9b}$$

If i deviates, it is not profitable to deviate to $q_i' < q_i^*$. Hence, the only profitable deviation is to $q_i'' > q_i^*$. Given i's deviation output exceeds j's, it follows that

$$R_i\left(q_j\right) \text{ given } q_i^* > q_j^* = \frac{(A - c_i - bq_j - c)}{2b}$$

And $q_i^* = \frac{2A-3c_i+c_j-3c}{6b}$, $p = \frac{2A+3c_i+3c+c_j}{6}$. The consistency condition (requiring that in equilibrium $q_i^* > q_j^*$) yields $-c_i - c + c_j > 0$. Hence, a sufficient condition for there not to exist a profitable deviation is that $c_j > c_i + c$.

The deviation profit is then given by:

$$\pi_{i,q_i>q_j} = \left(\frac{2A-3c_i+c_j-3c}{6b}\right)\left(\frac{2A-3c_i+3c+c_j}{6}\right) - c\left(\frac{-c_i-c+c_j}{6b}\right) =$$

$$= \frac{-12Ac_i+4Ac_j+6cc_i-6cc_j+9c_i^2+c_j^2-6c_ic_j+4A^2-3c^2}{36b}$$

The deviation is not profitable if and only if:

$$-9Ac_i+3Ac_j+6cc_i-6cc_j+9c_i^2+3c_j^2-9c_ic_j+3A^2-3c^2<0$$

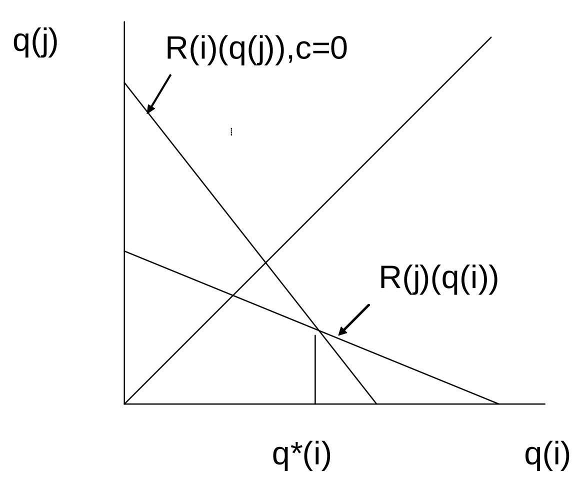
The deviation is never profitable in this environment ■

In case $c_j < c_i + c$, there are both the equilibria described above.

The intuition behind the equilibria is that, both if $q_i^{*UC} > q_j^{*UC}$ and if $q_j^{*UC} > q_i^{*UC}$, the condition for the existence of the equilibria is that firm i finds it efficient or inefficient to mimick firm j.

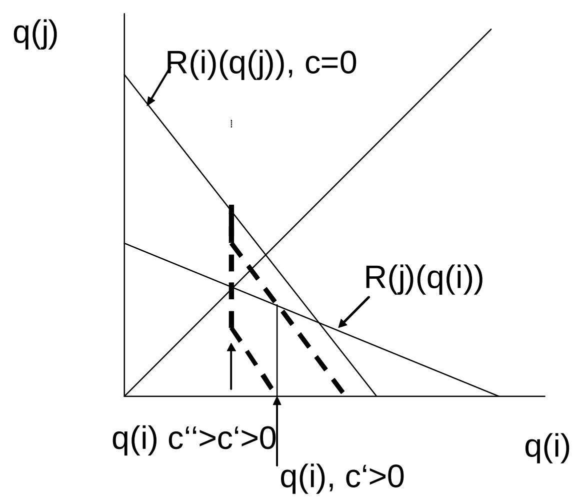
For a graphical intuition for the results, consider the following reaction function, in which the transport cost is null:

Figure 2: Reaction functions, null transport cost



In that case, the most efficient firm is producing more than the least efficient. As the transport cost increases, the quantity produced by the most efficient firm decreases, until, for a sufficiently high transportation cost, it hits the output of the least efficient firm. The following graph describes this situation:

Figure 3: Reaction functions with different transport costs



The previous graph displays the reaction functions of the two firms, and shows that, for values of c such that $c_i + c > c_j$ (represented by c' in the graph), the equilibrium prescribes $q_i^* = q_j^*$. This equilibrium is the same that would prevail if both firms had the same cost as the inefficient one.

If $c_i + c < c_j$ then $q_i^* > q_j^*$, the equilibrium is the same that would prevail if i's cost function were $c + c_i$.

2.3 The joint profit maximization in duopoly

Given the positive externality from an increase in rival's production enjoyed by the most efficient firm, there may be scope for efficiency in a cooperative game, in which a single decision maker maximizes the joint profit of the two firms.

When the positive externality offsets the standard monopoly inefficiency (with respect to the situation of duopoly), joint profit maximization yields a more efficient outcome than duopoly.

Joint profit maximization entails a market-level cost minimization. Any level of output is produced using the most efficient resources available on the market. Hence, the aggregate cost function employed in the social welfare maximization is the relevant cost function. Denote Q^{JM} the level of ouptut produced under joint profit maximization.

Denote $\Delta c = c_j - c_i$ as the marginal cost difference between the most efficient and the least efficient firm. Notice that, in terms of Δc, the assumption that both firms are producing a positive output in equilibrium determines $\Delta c < A + c - c_i$.

We now characterize conditions under which joint profit maximization performs better than standard competition in terms of efficiency.

Lemma 6 *Joint profit maximization is more efficient than Cournot competition:*
- *when* $c > \Delta c$, *if* $3\Delta c > 2(A - c_j)$;
- *when* $\frac{\Delta c}{2} < c < \Delta c$, *if* $4c - \Delta c_i > 2(A - c_j)$

Proof. The decision maker maximizes:

$$\max_Q Q\left((A - bQ) - \min\left(\frac{c_i + c_j}{2}, c_i + c\right)\right)$$

First order conditions are:

$$(A - 2bQ) = \min\left(\frac{\Delta}{2}, c_i + c\right)$$

Second order conditions are always satisfied.

At the optimum, it has to be:

$$Q^{JM} = \frac{A - \min\left(\frac{c_i+c_j}{2}, c_i + c\right)}{2b}$$

$$Q^{JM}_{\frac{c_i+c_j}{2} > c_i + c} = \frac{A - c_i - c}{2b}$$

$$Q^{JM}_{\frac{c_i+c_j}{2} < c_i + c} = \frac{A - \frac{c_i+c_j}{2}}{2b}$$

where Q^{JM} denotes the optimal output in the joint profit maximization. Output in the cooperative game exceeds the one in the competitive game if:

$$\begin{cases} Q^{JM}_{\frac{c_i+c_j}{2}<c_i+c} = \frac{A-\frac{c_i+c_j}{2}}{2b} \\ Q^{JM}_{\frac{c_i+c_j}{2}>c_i+c} = \frac{A-c_i-c}{2b} \end{cases} > \begin{cases} Q^{C}_{c_j>c_i+c} = \frac{2A-c_i-c_j-c}{3b} \\ Q^{C}_{c_j<c_i+c} = \frac{2A-2c_j}{3b} \end{cases}$$

■

Proposition 7 *Given the demand parameters, and the individual marginal costs, the increase in consumer welfare from joint profit maximization increases with c if* $\frac{\Delta c}{2} < c < \Delta c$*, while it decreases with c if* $c < \frac{\Delta c}{2}$*, and it remains constant if* $c > \Delta c$*;*

Given the demand parameter, the transportation cost, and the individual cost of the inefficient firm, consumer welfare increase from joint profit maximization increases with Δc *if* $c > \Delta c$*, while it decreases with* Δc *if* $\frac{\Delta c}{2} < c < \Delta c$*.*

Proof. It follows directly from computations from the previous results ■

When the equilibrium is symmetric in both cases, the externality is increasing with the asymmetry, hence the benefit of internalizing it also increases with the asymmetry. On the other hand, when the equilibrium is symmetric only in JM, while it is asymmetric in Cournot, the increase in c_i decreases cost by c_i in the duopoly, while it decreases overall cost only by $\frac{c_i}{2}$ in the JM. Hence, when this is the case, asymmetry increases Cournot output more than it does the JM output.

A similar intuition may be provided for the comparative statics on the transport cost. If both JM and UC are symmetric, then the outcomes in each case will depend only on the level of cost asymmetricity, not on the transport cost. An increase in the transport cost, up to the level in which they stop being symmetric, does not affect the cost function of each of the two. However, when JM is symmetric, an increase in transport cost still does not affect the JPM equilibrium, while it affects the UR (asymmetric) equilibrium.

2.4 Optimal allocation in duopoly

We now examine how the regulator should optimally allocate the transport capacity. The timing of the game is the following:

1) The regulator announces the allocation scheme of the cost reimbursement;

2) Firms compete in quantity;

3) The output is realized, and reimbursement takes place.

The cost function, after the allocation of transmission capacity, for firm i is given by:

$$C(q_i) = \begin{cases} c_i q_i + c(q_i - q_j) - t_i(q_i, q_j) \;\; if \;\; q_i > q_j \\ c_i q_i \;\; otherwise \end{cases}$$

Denote $\widehat{q}_i(q_j)$ and $\widehat{q}_j(q_i)$ the best response by i (respectively j) to the rival's output when i (respectively, j) is allocated the full transmission capacity, given by $t_i(q_i, q_j) = \min\left(c\left(q_i^* - q_j^*\right), t\right)$. On the other hand, denote $\widetilde{q}_i(q_j)$ and $\widetilde{q}_j(q_i)$ the best response by i (respectively, j) to the rival's output when i (respectively, j) is not allocated the transmission capacity. Finally, denote q_i^* and q_j^* the equilibrium output in the overall game.

First, we establish that the least efficient firm is indifferent to the reimbursement it receives; hence, the reimbursement for the least efficient firm has no effect for the aggregate output produced.

Lemma 8 $t_j(q_i, q_j)$ *is irrelevant in determining the outcome of the game*

Proof. As previously shown, in Cournot competition without the regulator's reimbursement, it has to be $q_j^{*UR} = R_j\left(q_i^{UR}\right) \leq q_i^{*UR}$, where $R_j\left(q_i^{UR}\right)$ is the reaction function of firm j to the output played by i in the unregulated equilibrium, $R_i\left(q_i^{UR}\right)$. At q_j^{*UR} firm j's cost is $C(q_j) = c_j q_j$, and no transportation cost is incurred by firm i. Firm i, on the other hand, incurs transportation cost $c\left(q_i^{*UR} - q_j^{*UR}\right)$.

When the regulator allocates transportation capacity, it reduces firm i's cost. However, even when j is entirely allocated the transport capacity, for any $q_i = q_j$, $C(q_i) \leq C(q_j) \Leftrightarrow q_j \leq q_i$. Given that, in equilibrium, i's output exceeds $j's$ output, it follows that firm j will not need the reimbursement, hence its allocation is irrelevant ∎

Under symmetry, Cournot profit from an asymmetric outcome decreases even when the transport cost is entirely reimbursed. Hence, no firm has an incentive to deviate from the Cournot outcome. Under the symmetric case, firms do not need the interconnection capacity.

Proposition 9 *If* $q_i = q_j$*, then both* $t_j(q_i, q_j)$ *and* $t_i(q_i, q_j)$ *are irrelevant in determining the outcome of the game. The regulator cannot take any effective actions in the market.*

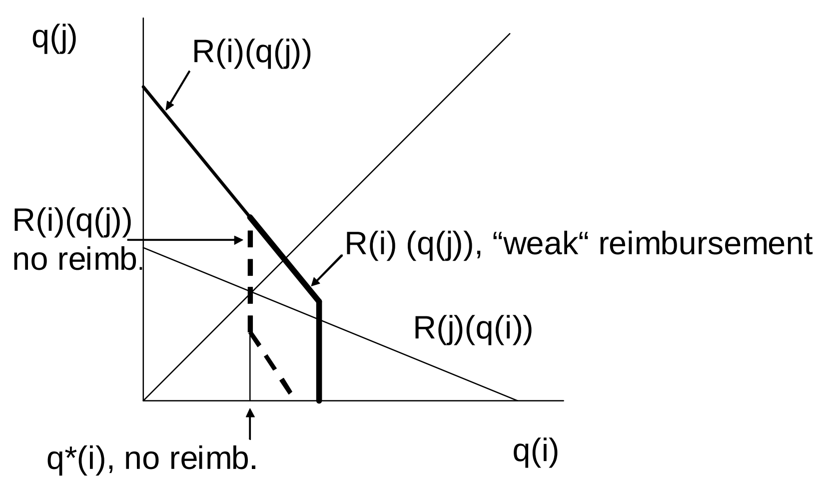
Proof. In Cournot equilibrium, $R_i(q_j) = q_j$ and $R_j(q_i) = q_i$. Hence, firms do not need transportation capacity ■

Hence, under symmetric cost, the regulator cannot modify the outcome of the game.

However, if costs differ between the two firms, even assuming firms compete à la Cournot, the regulator can increase market efficiency in two ways.

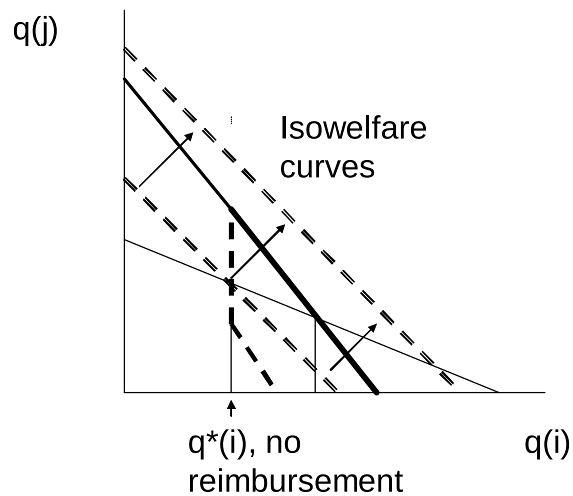
In a *weak* way, the regulator may reimburse the input (transportation) cost of the most efficient firm. In the aftermath of the reimbursement, the efficient firm faces a different cost structure, and optimizes accordingly. Notice that cost reimbursement is bounded by the resources available to the regulator.

Figure 4: Reaction functions with and without reimbursement



As a consequence of the regulator's reimbursement, the efficient firm decreases its cost, hence aggregate output increases, as well as welfare, as shown in the following picture:

Figure 4 bis: Reaction Functions and Welfare



In a *strong* sense, through an appropriate allocation of the transport capacity, a perfectly informed regulator may structure the reimbursement in such a way that the most efficient firm produces a higher output than that of the unregulated situation. The exact process through which that happens will be clear from what follows. In order to maximize social efficiency only through the capacity allocation, the regulator has to induce a cost function of the following form

$$C(q_i) = \begin{cases} c_i q_i \ for \ q_i \leq q_j \\ c_i q_i + (c + c_i)(q_i - q_j) \ \ for \ \overline{q} \geq q_i > q_j \\ c_i q_i \ for \ q_i > \overline{q} \end{cases}$$

Perfectly informed regulator Preliminarily to obtaining the results, we derive some general properties. The first states that overall welfare increases with q_i^* until aggregate output played by the two firms hits social optimum.

Lemma 10 $\frac{dS}{dq_i} > 0$ *if* $q_i + \widetilde{q}_j(q_i) \leq Q^{eff}$, $\frac{dS}{dq_i} < 0$ *otherwise, where* Q^{eff} *denotes the previously characterized benchmark of first best social efficiency.*

Proof. In any equilibrium of the game, the inefficient firm plays a lower output than the efficient firm, thus it bears a null cost for the transportation capacity, hence it is not affected by the allocation procedure. It follows that $q_j^* = \widetilde{q}_j$. In a linear demand system, as long as $q_i' > q_i$, $q_i' + R_j(q_i') > q_i + R_j(q_i)$. The fact that $q_j^* = \widetilde{q}_j(q_i)$ guarantees that maximization of social welfare entails picking two different values of q on the same reaction function. ■

In the current game, we are assuming that the regulator has a fixed amount of cost reimbursement, which we consider as sunk cost. Hence, the regulator derives no direct advantages from withholding part of the allocation. Denote as $t_i^*(q_i, q_j)$ the optimal (first best) allocation, and denote as q_i^{FI}, q_j^{FI} the equilibrium output under the full information regulator's optimal reimbursement scheme. Hence, we can derive the following:

Proposition 11 *A perfectly informed regulator may induce the most competitive firm to produce*

$$q_i^{*FI} = \begin{cases} \frac{2A+4c_j-6c_i}{3b} \ if \ c_j < c_i + c \\ \frac{-c_i + \frac{c_j}{2} + \frac{A}{2} + \frac{\sqrt{4c_i^2 - 4c_ic_j - 4Ac_i - 55c_j^2 + 2Ac_j + A^2 - 88cc_i + 80cc_j + 8Ac)}}{6}}{b} \ if \ c_j > c_i + c \end{cases}$$

by offering the following reimbursement scheme: $t_i^*(q_i, q_j) = \begin{cases} t \ for \ q_i \geq q_i^* - \epsilon \\ 0 \ for \ q_i < q_i^* - \epsilon \end{cases}$ *if* $t \geq q_i^* - q_i^U$.

If $t \geq q_i^* - q_i^U$*, the most competitive firm will be induced to produce* q_i^{**FI} *such that* $q_i^{**}\left(A - bq_i^{**} - \frac{A-c_j-bq_i^{**}}{2} - c_i - \left(q_i^{**} - \frac{A-c_j-bq_i^{**}}{2} - t\right)c\right) = \pi^U$

Proof. Solving explicitly for the optimal allocation scheme, we have to differentiate two cases:

- If $c_j < c_i + c$, then in equilibrium $q_i^{UC} = q_j^{UC} = \frac{(A-c_j)}{3b}$, and $p = \frac{(A+2c_j)}{3}$.

It follows that

$$\begin{aligned}\pi_i^{UC} &= \frac{(A-c_j)}{3b}\frac{(A+2c_j-3c_i)}{3} = \\ &= \frac{\left(A^2+Ac_j-3Ac_i-2c_j^2+3c_ic_j\right)}{9b}\end{aligned}$$

If allocated the capacity regardless of the output, firm i will optimally produce:

$$\begin{aligned}\widehat{q}_i^* &= \frac{A-2c_i+c_j}{3b} \\ R_j^*\left(\widehat{q}_i^*\right) &= \frac{A+c_i-2c_j}{3b} \\ Q &= \frac{2A-c_i-c_j}{3b}\end{aligned}$$

Aggregate output in this case is given by:

$$\frac{2A-c_i-c_j}{3b} > \frac{2A-2c_j}{3b}$$

The difference $\frac{2A-c_i-c_j}{3b} - \frac{2A-2c_j}{3b}$ represents the weak increase due to the cost effect of the capacity allocation. Price after the weak increase is given by:

$p = \frac{A+c_i+c_j}{3}$. Profit after it equals

$$\begin{aligned}\pi_i &= \left(\frac{A-2c_i+c_j}{3b}\right)\left(\frac{A-2c_i+c_j}{3}\right) \\ \pi_i &= \frac{A^2+4c_i^2+c_j^2-4Ac_i+2Ac_j-4c_ic_j}{9b}\end{aligned}$$

The difference in profit between the two scenarios is:

$$\Delta\pi = \frac{4c_i^2+3c_j^2-Ac_i+Ac_j-7c_ic_j}{9b}$$

Then, q_i^{FI} will solve the following equation:

$$q_i^{FI}\left(A-bq_i^{FI}-b\frac{(A-c_j-bq_i)}{2b}-c_i\right) = \frac{\left(A^2+Ac_j-3Ac_i-2c_j^2+3c_ic_j\right)}{9b}$$

The solution is:

$$\frac{A-c_j}{3b}<q_i^{FI}<\frac{2A+4c_j-6c_i}{3b}$$

It follows that, as long as $c_i<c_j+c$,

$$q_i^{FI}=\frac{2A+4c_j-6c_i}{3b}$$
$$q_j^{FI}=\frac{A-7c_j+6c_i}{6b}$$
$$Q^{FI}=\frac{5A+c_j-6c_i}{6b}$$

It is now easy to show that a perfectly informed regulator, by awarding $t_i^*(q_i)=0$ for $q_i<\frac{2A+4c_j-6c_i}{3b}-\epsilon$, $t_i^*(q_i,q_j)=t$ for $q_i\geq\frac{2A+4c_j-6c_i}{3b}+\epsilon$, can implement Q^{FI}.

- If $c_j>c_i+c$, then in equilibrium we have $q_i^{UC}=\frac{A-2c_i+c_j-2c}{3b}$, $q_j^{UC}=\frac{A+c_i-2c_j+c}{3b}$, $p^{UC}=\frac{A+c_i+c+c_j}{3}$. Hence, $q_i^{UC}-q_j^{UC}=\frac{-c_i+c_j-c}{3b}$

$$\pi_i^U=\left(\frac{A-2c_i+c_j-2c}{3b}\right)\left(\frac{A-2c_i+c_j+c}{3}\right)-c\left(\frac{c_j-c_i-c}{b}\right)=$$
$$=\frac{-4Ac_i+2Ac_j+11cc_i-10cc_j+4c_i^2+c_j^2-4c_ic_j-Ac+A^2+7c^2}{9b}$$

q_i^{FI} solves the following equation:

$$q_i^{FI}\left(A-bq_i^{FI}-b\frac{(A-c_j-bq_i)}{2b}-c_i\right)=$$
$$=\frac{-4Ac_i+2Ac_j+11cc_i-10cc_j+4c_i^2+c_j^2-4c_ic_j-Ac+A^2+7c^2}{9b}$$
$$q=\frac{-c_i+\frac{c_j}{2}+\frac{A}{2}+\frac{\sqrt{4c_i^2-4c_ic_j-4Ac_i-55c_j^2+2Ac_j+A^2-88cc_i+80cc_j+8Ac)}}{6}}{b}$$

Even in this case, the allocation is easily implementable by the regulator.

Finally, if the reimbursement in the hands of the regulator is not sufficient to fully cover the transportation cost of the efficient firm, then the most efficient firm has to pay part of the transport cost, and q_i^{FI} solves the following equation:

$$q_i^{**}\left(A - bq_i^{**} - \frac{A - c_j - bq_i^{**}}{2} - c_i - \left(q_i^{**} - \frac{A - c_j - bq_i^{**}}{2} - t\right)c\right) = 0$$

■

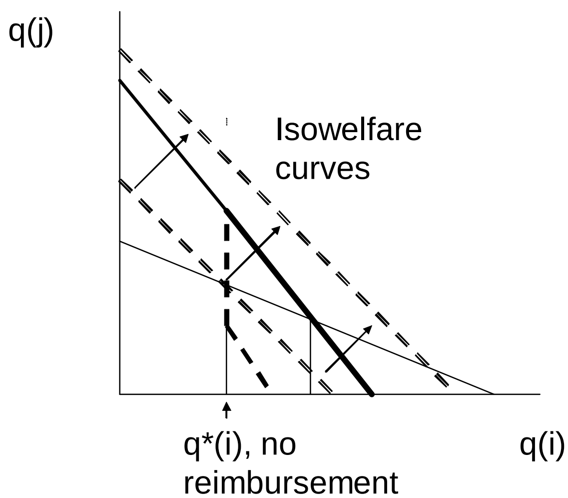
The intuition behind the result is the following. *Given* that i has been awarded the available transmission capacity, $\frac{\partial \pi}{\partial q} > 0$, $\widetilde{q}_i \leq q \leq \widehat{q}_i$, $\frac{\partial \pi}{\partial q} < 0$, $q \leq \widehat{q}_i$. At $\widehat{q}_i(q_j)$, the best response given that the reimbursement has been allocated to i, firm i would maximize profit given the transportation allocation. At the optimal point, output is increased with respect to the unregulated benchmark, in which no firm is provided with transmission capacity, and so is i's profit. However, social efficiency is increased by further increasing output beyond the optimal point. By denying the reimbursement unless $q_i^* > \widehat{q}_i$, i.e, the equilibrium output is *above* the individually optimal point given the reimbursement rules, the regulator can increase social welfare. The reason why this is feasible for the regulator is that this reimbursement rule makes $q' = \widehat{q}_i(q_j)$ - the optimal point given the allocation of transport capacity - an unfeasible alternative, since at q' i is not allocated the reimbursement; hence, it can only deviate to $\widetilde{q}_i(q_j)$ - the optimal point given no allocation (which yields a lower profit). The regulator exploits the difference

$$\pi(\widehat{q}_i(q_j)) - \pi(\widetilde{q}_i(q_j)) = \delta$$

to induce i to play $q_i^* > \widehat{q}_i(q_j)$ until $\pi(\widehat{q}_i(q_j)) - \pi(q_i^*) = \delta$ (if $\widetilde{q}_i(q_i^*) + q_i^* < Q^{eff}$

The following picture identifies the isoprofit for firm i and the isowelfare curves:

Figure 5: Reaction functions, isowelfare, and isoprofit



The regulator attains the equilibrium point at which, within the set of allocations that guarantee to the firm a payoff at least as high as in the case without reimbursement, guarantees to firm i the same profit it would get without reimbursement

Notice that, in this case, the regulator withdraws some of the interconnection capacity for certain out-of-equilibrium values in order to align the efficient firm's behavior with a socially desirable one. In the result, the outside option for the players not to be subject to any regulatory constraint by withdrawing the option of receiving the transmission capacity allocation plays a fundamental role.

Inefficiency in the unregulated market is due both to the standard market power inefficiencies related to Cournot duopoly, and to the cost-externality of the inefficient firm on the efficient one. The regulator cannot directly solve the externality (this would entail increasing the production of the inefficient firm); however, its reimbursement may mitigate it, or even indirectly solve it (as long as the sum to be allocated is sufficient). By mitigating the externality, the regulator increases the profit of the efficient firm. The regulator can appropriate the gain in profit obtained by the most efficient firm in the aftermath of the reimbursement, through the threat of not awarding such reimbursement. The regulator "transforms" the appropriation of profit into appropriation of consumer welfare, by requiring a higher production of the efficient firm.

The increase in output hence depends on the benefits induced by the regulator in alleviating the externality. When the externality does not exist (in the case of equal efficiency), then the regulator's reimbursement is of no use, hence it cannot intervene in the market.

For example, as long as $c_j < c_i + c$, the price under full information is $p^{FI} = A - bQ^{FI} = \frac{A}{6} - \frac{c_j}{6} + c_i$. Since marginal cost is given by $c_i + c$, a sufficient condition for Q^{FI} to remain unmodified under the assumption that the regulator has to build the transmission line for the market is the following: $p^{FI} < MC$, or $A > 6c + c_j$. Under this assumption, the marginal benefit of the addition in transmission capacity exceeds its marginal cost, which includes also the building cost[2]. Hence, the regulator's direct intervention in the market is (weakly) welfare-maximizing, with respect to standard nodal pricing, even if the regulator has to build the transmission capacity before allocating it. To grasp an intuition for the result, consider that the reimbursement may be regarded as a subsidy, and it is well known that subsidies may be welfare-enhancing under imperfect competition.

As a last comment, we notice that, in this environment, the effectiveness of the regulator's intervention crucially depends on the asymmetry in the market. In a very asymmetric market, the scope for the regulatory intervention is broader than in a symmetric market.

3 Duopoly in each market

3.1 The unregulated benchmark

Assume now a different market structure. Two firms produce in each node. The two producers located on the same node have exactly the same production cost function, but production costs of producers located in different

[2]This is true under the assumption that public funds are costless to the regulator

nodes may differ. Given that the transportation constraint is net, the transmission cost is paid only by firms located in the node in which the highest aggregate output is produced. It is still the case (and will be formally proved in what follows) that firms in the least efficient node are producing a lower output than firms located in the most efficient one, hence only the most efficient firms, denoted i, will have to pay the transportation cost. Each firm pays a transportation cost equal to what I believe to be the most appropriate measure of the marginal contribution firm i_1 makes to the total transportation cost, given by $\max\left(\left(q_i^1 - \frac{q_j^1+q_j^2}{2}\right)c, 0\right)$. With this formulation, firm i^1 does not pay any cost, if $2q_i^2 = q_j^1 + q_j^2$. In words, firm i^1 is not paying any costs if, assuming i^2 produced exactly as much as i^1, the i node would produce as much as the j node, and thus it would not pay any interconnection costs. For each additional unit with respect to the one that, if exactly matched by i^2, would entail no transportation cost for node i, then firm i^1 is paying the transport cost c. The same exact argument holds for firm i^2.

In the stylized model, assume there are two exactly symmetric firms i, denoted i^1 and i^2, with production cost c_i, and two firms j, denoted j^1 and j^2, whose production cost is c_j. Following the previously expressed argument, the transport cost is given by $C\left(q_i^1\right) = \max\left(\left(q_i^1 - \frac{q_j^1+q_j^2}{2}\right)c, 0\right)$.

The incentives involved in the current unregulated game are the same as in the two–monopolies game, with the addition of the standard incentives involved in the Cournout competition with substitute products.

First, we explicitly characterize the Cournot equilibrium of the unregulated game without reimbursement.

Firm $i^{1(2)}$ is maximizing:

$$\max_{q_i^{1(2)}} \left(A - bq_i^1 - bq_i^2 - bq_j^1 - bq_j^2\right) q_i^{1(2)} - c_i q_i^{1(2)} - \max\left(\left(q_i^1 - \frac{q_j^1+q_j^2}{2}\right)c, 0\right)$$

Firm $j^{1(2)}$, in its turn, is maximizing:

$$\max_{q_j^{1(2)}} \left(A - bq_i^1 - bq_i^2 - bq_j^1 - bq_j^2\right) q_j^{1(2)} - c_j q_j^{1(2)}$$

The reaction functions of the two firms are portrayed in what follows:

$$R_{i^1}\left(q_i^2, q_j^1, q_j^2\right) = \begin{cases} q_j \ if \ \pi_{i,q_i^{1*}=q_j^{1*}} \geq \pi_{i,q_i^{1*}>q_j^{1*}} \\ \max\left(\frac{\left(A-c_j-c-2bq_j-bq_i^2\right)}{2b}, q_j\right) \ if \ \pi_{i,q_i^{1*}=q_j^{1*}} \leq \pi_{i,q_i^{1*}>q_j^{1*}} \end{cases}$$

$$R_{i^2}\left(q_i^1, q_j^1, q_j^2\right) = \begin{cases} q_j \ if \ \pi_{i,q_i^{2*}=q_j^{1*}} \geq \pi_{i,q_i^{2*}>q_j^{2*}} \\ \max\left(\frac{\left(A-c_j-c-2bq_j-bq_i^1\right)}{2b}, q_j\right) \ if \ \pi_{i,q_i^{2*}=q_j^{1*}} \leq \pi_{i,q_i^{2*}>q_j^{2*}} \end{cases}$$

$$R_{j^1}\left(q_i^1, q_i^2, q_j^2\right) = R_{j^2}\left(q_i^1, q_i^2, q_j^1\right) = \frac{A - c_j - bq_i^1 - bq_i^2}{3b}$$

We will distinguish the equilibria that can possibly emerge in this game into three kinds of equilibria:

-the fully symmetric equilibrium, in which $q_i^{1*} = q_i^{2*} = q_j^{1*} = q_j^{2*}$;

-the partially symmetric equilibrium, in which firms having the same efficiency produce the same output, $q_i^{1*} = q_i^{2*} \neq q_j^{1*} = q_j^{2*}$;

-the asymmetric equilibrium, in which $q_i^{1*} = q_i^{2*} = q_j^{1*} \neq q_j^{2*}$

Lemma 12 *If $c_j \leq c_i + c$, then a symmetric equilibrium exists, in which:* $q_i^{1*} = q_i^{2*} = q_j^{1*} = q_j^{2*}$

Proof. In a symmetric equilibrium, it has to be that:

$$q_j^{1*} = q_j^{2*} = q_i^{1*} = q_i^{2*} = \frac{A - c_j}{5b}$$

Assume $q_i^{2*} + q_j^{1*} + q_j^{2*} = 3q_j^{1*}$, Then, it must be that

$$q_j^1 = q_j^2 = q_i^2 = \frac{A - bq_i^1 - c_j}{4b}$$

Firm i's reaction is the following:

$$R_{i^1}\left(q_i^2, q_j^1, q_j^2\right) = \begin{cases} \frac{A-bq_i^1-c_j}{4b} \ if \ \pi_{i,q_i^{1*}=q_j^{1*}} \geq \pi_{i,q_i^{1*}>q_j^{1*}} \\ \max\left(\frac{(A-c_i-c-3bq_j)}{2b}, q_j + \epsilon\right) \ if \ \pi_{i,q_i^{1*}=q_j^{1*}} \leq \pi_{i,q_i^{1*}>q_j^{1*}} \end{cases}$$

In equilibrium with $q_i^* > q_j^*$, it has to be that $q_j^* = \frac{A+c_i+c-2c_j}{5b}$, and $q_i^{1*} = \frac{A-4c_i-4c+3c_j}{5b}$. The consistency condition in the previous equilibrium with $q_i^{1*} > q_j^{1*}$ requires $c_j > c_i + c$. Hence, if the consistency condition is not holding, or

$$c_j < c_i + c$$

the symmetric equilibrium may emerge as the outcome of the game. ■

We now pass to examine the partially symmetric equilibrium.

Lemma 13 *If $c_j > c_i + c$, then a partially symmetric equilibrium exists, in which: $q_i^{1*} = q_i^{2*} > q_j^{1*} = q_j^{2*}$*

Proof. The partially symmetric equilibrium is given by: $q_i^1 = q_i^2 \neq q_j^1 = q_j^2$. In this equilibrium, the reaction functions are:

$$q_j^1 = q_j^2 = \frac{A - 2bq_i^1 - c_j}{3b}$$

$$q_i^1 = q_i^2 = \frac{A - 2bq_j^1 - c_i - c}{3b}$$

The efficient firms play:

$$q_i = \frac{A - 3c_i - 3c + 2c_j}{5b}$$

$$q_j = \frac{A + 2c_i + 2c - 3c_j}{5b}$$

In this equilibrium, we have $q_j^* = \frac{A-3c_j+2c_i+2c}{5b}$, and $q_i^* = \frac{A+2c_j-3c_i-3c}{5b}$. The consistency condition in the previous equilibrium with $q_i^{1*} > q_j^{1*}$ requires

$$c_j > c_i + c$$

■

We now state the general condition for the existence of the equilibrium

Proposition 14 *If $c_j > c_i + c$, there exists a unique equilibrium, in which $q_i^{1*} = q_i^{2*} > q_j^{1*} = q_j^{2*}$;*

If $c_j \leq c_i + c$, there exists a unique equilibrium, in which: $q_i^{1*} = q_i^{2*} = q_j^{1*} = q_j^{2*}$

No other equilibria exist

3.2 Duopoly in each market: a perfectly informed regulator

A perfectly informed regulator aims at maximizing overall welfare.

In any equilibrium, the inefficient firms, which do not pay the transportation cost, and hence are not interested in the regulator's reimbursement, play

$$q_j^1 = q_j^2 = \frac{A - bq_i^1 - bq_i^2 - c_j}{3b}$$

Clearly, social welfare increases with q_i^1 and q_i^2. Also, given the symmetric structure of the firms involved, optimal allocation by a perfectly informed regulator involves $q_i^1 = q_i^2$

We look for a unique Nash equilibrium of a game in which at first the regulator allocates the reimbursement criterion, then firms compete à la Cournot, and finally, production takes place.

For the equilibrium to be unique, it has to be that π_i given that i is allocated the reimbursement equals the profit i would get without the reimbursement being obtained; on the other hand, the profit firm i gets given that it is allocated the reimbursement equals the profit i would get with the reimbursement being obtained. The profit emerging from the equilibrium without reimbursements is the following.

If $c_j \leq c_i + c$, then the equilibrium in the absence of reimbursement prescribes $q_i^{1*} = q_i^{2*} = q_j^{1*} = q_j^{2*} = \frac{A-c_j}{5b}$, and $p = A - 4b\frac{A-c_j}{5b} = \frac{A+c_j}{5}$

$$\pi_i^1 = \pi_i^2 = \left(\frac{A+c_j}{5}\right)\left(\frac{A-c_j}{5b} - c_i\right) =$$

$$= \left(\frac{A+c_j}{5}\right)\left(\frac{A-c_j-5bc_i}{5b}\right) = \frac{(A+c_j)(A-c_j-5bc_i)}{25b}$$

In equilibrium, it has to be that

$$p^* q_i^* - q_i^* c_i = \frac{(A+c_j)(A-c_j-5bc_i)}{25b}$$

and

$$p^* = A - b\left(2q_i^* - 2\frac{A-2bq_i^*-c_j}{3b}\right)$$

.

Hence, the regulator solves the following equation:

$$q_i^*\left(A - b\left(2q_i^* - 2\frac{A-2bq_i^*-c_j}{3b}\right) - c_i\right) = \frac{(A+c_j)(A-c_j-5bc_i)}{25b}$$

$$q_i^* = \frac{\left(\frac{2c_j+3c_i-5A}{3}\right) \pm \sqrt{\frac{101A^2+19c_j^2+45c_i^2-100Ac_j-90Ac_i-5Abc_j-5bc_ic_j}{45}}}{\frac{-20b}{3}}$$

We have thus proved the following:

Proposition 15 *A perfectly informed regulator may induce the most efficient firms to produce*

$$q_i^* = \begin{cases} \frac{\left(\frac{2c_j+3c_i-5A}{3}\right)\pm\sqrt{\frac{101A^2+19c_j^2+45c_i^2-100Ac_j-90Ac_i-5Abc_j-5bc_ic_j}{45}}}{\frac{-20b}{3}} & if\ c_j < c_i + c \\ \frac{-c_i+\frac{c_j}{2}+\frac{A}{2}+\frac{\sqrt{4c_i^2-4c_ic_j-4Ac_i-55c_j^2+2Ac_j+A^2-88cc_i+80cc_j+8Ac)}}{6}}{b} & if\ c_j > c_i + c \end{cases}$$

by offering the following reimbursement scheme:

$$t_i^*\left(q_i, q_j\right) = \begin{cases} t\ for\ q_i \geq q_i^* - \epsilon \\ 0\ for\ \ q_i < q_i^* - \epsilon \end{cases}$$

if $t \geq q_i^* - q_i^U$.

Notice that, in the duopoly case, the cost externality is more significant than in the monopoly case (the difference in output between the efficient and the inefficient firms is larger under duopoly than under monopoly, for a given c_i and c_j; hence, the regulator, for a given c_i and c_j, has more room for intervening in the market under duopoly than under monopoly.

4 Conclusions

The paper shows that, when the interconnection cost is paid only by the node in which export exceeds import for an amount equal to the net flow, the equilibrium under nodal pricing (the prevailing market design in the electiricity industry) may prescribe an inefficiently low output. This is due to the positive externality that an increase in output in the import-node would generate on the cost function of the export node.

In this context, a direct intervention of the regulator in the allocation of transmission capacity may be beneficial to social welfare, both under the assumption that the trasmission line is available to the regulator at no cost, and under the assumption that the regulator has to build it. The regulator's intervention in allocating the transmission capacity cannot directly solve the externality, since the regulator's offer has no appeal on firms in the import node, which do not need interconnection capacity. However, by allocating the available interconnection capacity, the regulator achieves the same outcome as though the externality were solved. Indeed, it eliminates the extra cost on the excess export, by saving the most efficient firm the cost of building its interconnection infrastructure, meanwhile contributing to reducing the Cournot market power distortion. When firms are completely symmetric, the net-flow externality is absent.

Since the ability of the regulator to limit Cournot market power comes because of the benefits it provides firms in indirectly eliminating the net-flow externality, when such externality is absent, the regulator has no room for intervention even in limiting market power..

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